**Optimisation**

**Extended investigation Part 2:** **In-class validation**

**Solutions and marking key**

**Question 1**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (a) |  | * Identifies formula for volume | 1 |
| (b) |  | * Identifies expression with 3 dimensions * Substitutes another expression for length | 1  1 |
| (c) |  | * Identifies expression for volume | 1 |
| (d) |  | * Differentiates expression for volume * Applies linearity | 1  1 |
| (e) |  | * Equates derivative to zero * Solves resulting equation discarding zero result | 1  1 |
| (f) |  | * Substitutes w into expression for volume * Calculates volume and states units | 1  1 |
| (g) | Length is 70.22 cm  Height = 52.67 cm | * Determines length * Determines height | 1  1 |

**Question 2**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (a) |  | * Differentiates expression for volume * Applies linearity | 1  1 |
| (b) |  | * Equates derivative to zero * Solves resulting equation discarding zero result | 1  1 |
| (c) |  | * Calculates volume | 1 |
| (d) | Length is 70.22 cm  Height = 52.67 cm | * Determines length * Determines height | 1  1 |

**Question 3**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (a) |  | * Expresses relationship between radius or diameter and height * Determines *h* in terms of *r* * Substitutes expression for h in formula for volume of a cylinder | 1  1  1 |
| (b) |  | * Differentiates expression for volume * Applies linearity * Equates derivative to zero and determines value for radius | 1  1  1 |
| (c) | (i)  (ii)  Length = 158 – 2d = 52.67 cm | * Substitutes value for r into formula for volume * Calculates volume * Determines length | 1  1  1 |

**Question 4**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (a) |  | * Expresses relationship between *w, l* and *a* * Express *h* in terms of *a* * Substitutes expression for *h* in rule for volume | 1  1  1 |
| (b) |  | * Differentiates expression for volume * Applies linearity * Equates derivative to zero * Determines value for radius | 1  1  1  1 |

**Question 4 (cont’d)**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (c) | (i)  (ii)  Length = width = 2*a* = 52.67 cm  Height = 158 – length – width  Height = 52.67 cm | * Substitutes value for r into formula for volume * Calculates volume * Determines length, width and height | 1  1  2 |

**Part B**

**Question 1**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (a) | Graph the function for volume  Locate greatest value maximum value of “*y*” value | * Identifies alternate method * Describes method | 1  1 |
| (b) | Rate at which the volume changes with respect to one of the dimensions | * Describes changing rate * Identifies variables | 1  1 |
| (c) | The volume has stopped increasing | * Identifies zero rate of change | 1 |

**Question 2(a)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Solution   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Shape | Maximum volume  (cm3) | Dimensions for maximum volume  (cm) | | | |  |  | length | width | height | | Rectangular prism | 129 854 | 70.22 | 35.11 | 52.67 | | Triangular prism | 64 927 | 70.22 | 35.11 | 52.67 | | Cylinder | 114 735 | 52.67 | 52.67 | 52.67 | | Hexagonal prism | 94 885 | 52.67 | 52.67 | 52.67 | | |
| Marking key/mathematical behaviours | Marks |
| * Completes table with data obtained | 1 |

**Question 2**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Solution | Marking key/mathematical behaviours | Marks |
| (b) | Triangular prism  Hexagonal prism  Cylinder  Rectangular prism  No pattern apparent in the listing – number of sides is increasing but for the rectangular prism | * Ranks and makes appropriate comment | 1 |
| (c) | Height is always 52.67 cm  Height is one third of total linear measurement  Where there were no restrictions on length and width, they were equal and the shapes almost “cubic” | * Describes two aspects of the dimensions | 1  1 |
| (d) | Rectangular prism with *l = w = h* = 52.67  Volume = 146 113 cm3 | * Identifies shape and dimensions satisfying condition * Gives volume greater than those calculated | 1  1 |